Appendix 2: Volume of Gaussian Curve

Calculating the volume of revolution of the curve:

$$I_x = I_o e^{\left(-\frac{x^2}{2\sigma^2}\right)} . \quad \text{(A2.1)}$$

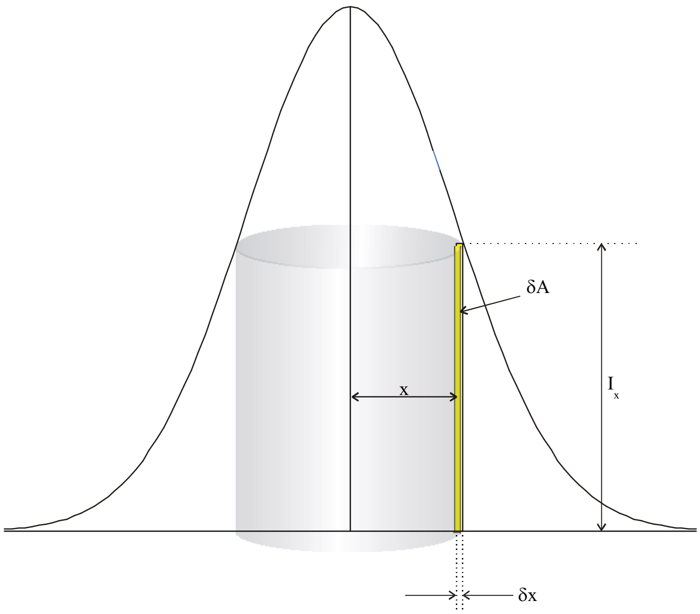

Figure 1: Calculation of volume integral of gaussian curve.

With reference to Figure 1, take a small strip of width dx and height I_x under the curve. The strip's area is therefore:

$$dA = I_x \, dx . \quad \text{(A2.2)}$$

By revolving this area about the y axis, a hollow cylinder is produced (Figure 1), whose volume is given by:

$$dV = 2\pi x \cdot dA = 2\pi x \, I_x \, dx . \quad \text{(A2.3)}$$

Substituting in the equation for I_x gives:

$$dV = 2\pi x \, I_o e^{\left(-\frac{x^2}{2\sigma^2}\right)} . \quad \text{(A2.4)}$$

Integrating from x = 0 to x = ∞ to obtain the total volume of revolution under the gaussian, V_g:

$$V_g = 2\pi I_o \int_0^\infty x\, e^{\left(-\frac{x^2}{2\sigma^2}\right)} dx \tag{A2.5}$$

$$= 2\pi I_o \left[-\sigma e^{\left(-\frac{x^2}{2\sigma^2}\right)} \right]_0^\infty \tag{A2.6}$$

$$= 2\pi I_o \sigma^2 \tag{A2.7}$$

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Now a top hat cylinder of radius r_c and equal height I_o has a volume V_c of:

$$V_c = \pi r_c^2 I_o , \tag{A2.8}$$

so that for the two volumes to be equal, we equate (A2.7) and (A2.8), and solving for r_c we obtain:

$$r_c = \sqrt{2}\ \sigma . \tag{A2.9}$$